Probabilistic autonomous navigation using Risk-RRT approach and models of human interaction

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***Abstract*— Autonomous transportation in human environments must follow social conventions. An autonomous wheelchair, for example, must respect proximity constraints but also respect people interacting, it should not break interaction between people talking, unless the user want to interact with them. In this case, the robot (i.e. the wheelchair) should find the best way to join the group. In this paper, we propose a risk-based navigation method which include risk of collision but also risk of disturbance. Results exhibit new emerging behavior showing how the robot takes into account social conventions in its navigation strategy.**

***Index Terms*— Proxemics, Human aware navigation, risk assessment.**

I. Introduction

Robots enter more and more into human environments. As areas of mobile service robotics and robotic assistance of humans are becoming more common in everyday life, humans need to share the physical space with robots and robots need to take into account the presence of humans. To be accepted, robots must behave in a socially acceptable way. Their trajectories must be safe but also predictable. Their behavior should follow social conventions, respecting proximity constraints, avoiding people interacting or joining a group engaged in conversation without disturbing.

People maintain a set of social conventions related to space when they are interacting for example in a conversation [1]. The sociology literature often refer to the concept of personal space proposed by Hall [2] which characterizes the space around a human being in terms of comfort to social activity. Concerning interactions between people, an o-space is described in the sociology literature. This space models casual conversations between people interacting [1]. The perception of the territorial boundaries established by group of humans and the respect of these bounds is an evidence of social behavior. Moreover transporting a human restrict us to respect social conventions when navigating, then if we want to develop social robots or wheelchair like robots, the notion of human-human interaction must be explicitly addressed.
The ideas presented in this paper will be implemented in a wheelchair to transport people with reduce mobility (PRM) in airports (fig. 1). Assistance to mobility in airports is a big challenge, in [3] many scenarios of application could be found mainly to restore to the users autonomy and privacy, for example, some PRMs reported that they felt abandoned in PRM areas in the departure lounge, and were worried that they might miss their flights. The interaction between the wheelchair and the user poses also many challenges due to the fact that passengers present varying disabilities (including visually impaired and deaf and hard of hearing), but a solution for this problem is not discussed in this paper.

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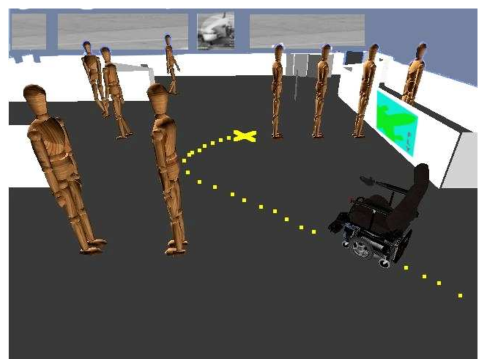


Fig. 1. Autonomous transport of people with reduced mobility in airports is a clear application of our strategy which takes in account human-human interaction. In this figure we can see a representation of the depart lounge of an airport, some social conventions could be observed, face to face interaction and staying in the queue. The wheelchair must navigate in the environment while respecting the cited conventions.

In this article, we propose a simple way to estimate the o-space in the case of two agents interacting based on their positions and orientations and propose an approach to take advantage of it in autonomous navigation of a wheelchair.

Section II proposes a state of the art of human aware navigation. Section III defines the concepts of spatial behavior and describes proxemics models used to take decisions in our navigation system. Section IV describes the algorithm of navigation called Risk-RTT and explains the extensions done. In section V the simulation of the navigation of an autonomous wheelchair in presence of humans interacting is presented. Section VI presents conclusions about the work and perspectives.

II. State of Art

Considering the literature, we can observe the growing interest of the robotics community in research that includes

the behavior of humans and its impact in the development of tasks by the robot. In [4] it is argued that moving in easily understood and predictable ways will both improve people's trusting and comfort with the robot as well as will help to insure the safety of people moving near the robot.
In [5] the authors propose a method for a robot to join a group of people engaged in conversation. The results of the implementation and the experiments conducted with their platform show a human-like behavior as judged by humans. Some approaches [6]–[11] have been conducted to establish the rules that probably will govern the physical behavior of robots regarding interaction with humans. Closer to human aware navigation and management of physical space, we could mention [12] where a motion planner is presented which takes explicitly into account its human partners. They introduce criteria based both on the control of the distance between the robot and the human, and on the control of the position of the robot in the human's field of view.
In [13] an adaptive system for detecting whether a person seeks to interact with the robot based on the person's pose and position is introduced. In [14] a framework for representing social conventions as components of a constraint optimization problem is presented. A* path planner is implemented with constraints like shortest distance, personal space and pass on the right.
Work presented in [15] propose Spatial Behavior Cognition Model (SBCM), a framework to describe the spatial effects existing between people and people, and people and environment, SBCM is used to learn and predict behaviors of pedestrians and for helping a service robot to take navigation decisions.
In almost all the cited works the concept of personal space is present but the concept of o-space and f-formations have not been included explicitly. We think these last concepts can give us a clue to consider the interactions between the dynamic obstacles in the environment and to improve the autonomous navigation by a better understanding of humans management of space.

III. Concepts of Social Behavior

To understand the perceived behaviors in human-human interaction and the resulting management of space, we can support us on the works developed in the area of sociology to define some concepts as personal space, o-space and F-formations.

A. Personal Space

The term Proxemics was proposed for Hall [2] to describe the use of space between humans, he observed the existence of some rules not written that conducted the people to keep distances from others, and others respect this space, he proposed that space around a person in social interaction is classified as follows:

- the public zone > 3.6m,
- the social zone > 1.2m
- the personal zone > 0.45m
- the intimate zone < 0.45m

That definition is important because it represents a useful tool for a robot to understand the intentions of the humans. It is well known that these measures are not strict and that they change depending on age, culture and type of relationship but the categories proposed explain very well reactions like the uncomfortable sense of a stranger invading your intimate zone or the perception of somebody looking social interaction because he is entering to your social zone. In general people are more strict regarding their frontal space. In the rest of the article we use personal space as synonymous of personal zone plus intimate zone.
The model that we have implemented to represent personal space is defined in [16], it consist in blending two Gaussian functions both of them centered in the position of the person. The first one represents the personal space situated in front of human and for this reason it is wider than the last one representing the back space. The figure 2 shows an example of personal space for two people walking, the measures are projected in the plane of floor, the values obtained from the gaussian are higher in the center than in the borders.

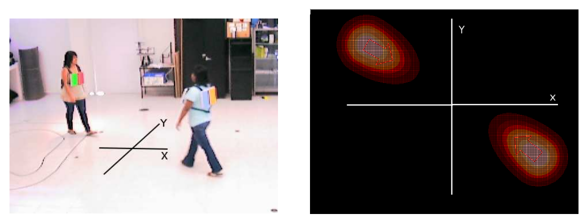


Fig. 2. Estimated personal space for two people that walk projected in the floor.

B. F-formations

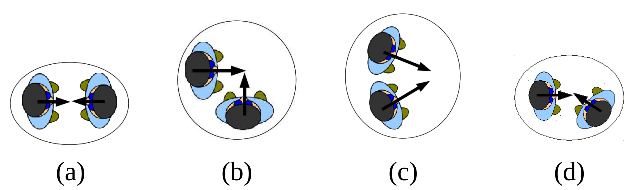


Fig. 3. Examples of F-formations: (a) Vis-a-vis, (b) L-Shape, (c) C-Shape, (d) V-Shape.

In [17] Kendon proposed that people interacting in groups follow some spatial patterns of arrangement. When people are executing some activity they claim an amount of space related to that activity, this space is respected by other people and Kendon referred it as individual's *transactional segment*. This transactional segment can vary depending on body size, posture, position and orientation during the activity. Moreover the groups can establish a joint or shared transactional segment and only the participants have permitted access to it, they protect it and others tend to respect it. The *o-space* is that shared transactional segment reserved for the main activity. This space is surrounded by a narrower one, called the *p-space*, which provides for the placement of the participant's bodies and also personal things. An *F-formation* system is the spatial-orientation arrangement that people create, share and maintain around their o-space. To become a member of a formation of this sort, you have to be in the p-space.

C. Model of o-space in F-formations

As there is not an exact physical definition of o-space we will describe in this section how we can estimate its location. When more than two people are in conversation they exhibit an F-formation with circular shape then the o-space could be taken as a circle whose center coincides with that of the inner space. In the case of two people some F-formations have been identified as the most frequents [1]. In our model, the o-space will be dependent from the particular F-formation identified: Vis-a-vis, L-Shape, C-Shape or V-Shape (fig. 3). The definition found in the reference mentioned before permits to get a geometric representation for each F-formation, based in the position and orientation of the body of participants.

Given the positions of pedestrians $H_1 = (x_1, y_1)$ and $H_2 = (x_2, y_2)$ in the plane of the floor and their respective orientations ϕ_1 and ϕ_2 around the normal to that plane, we calculate D_H as the euclidean distance between H_1 and H_2. We calculate also a point V_i as the intersection of the vectors beginning in H_1 and H_2 in the direction of ϕ_1 and ϕ_2, respectively. Let H_{12} be the mean point between H_1 and H_2. Let C be the mean point between V_i and H_{12}. Calculate D_i as the distance between V_i and H_{12}.

The o-space could be represented by a two-dimensional Gaussian function Γ_c of covariance matrix S and centered in C, then for each point Q around the center we have:

$$\Gamma_{C,S}(Q) = e^{-\frac{1}{2}(Q-C)^t S^{-1}(Q-C)} \tag{1}$$

where S is a diagonal covariance matrix defined as:

$$S = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix}. \tag{2}$$

To get the shape of the o-space in function of the F-formations, the values chosen for the parameters are $\sigma_x = D_H/4$ and $\sigma_y = D_i/2$. In the particular case of the Vis-a-Vis formation $\sigma_y = 0.6$. The orientation of the Gaussian is in the direction of the segment $\overrightarrow{H_{12}C}$, this coincides with the location of the point of interest of humans as exhibited by the orientation of their bodies.

The p-space is considered as the area between the border of the o-space and the same border enlarged by the average size of the humans in conversation. For effects of implementation o-space is discretized using a grid and taking the result of evaluating the center of each cell as the value for the cell. All the elements defined can be seen in fig. 4 for the case of an L-Shape F-formation.

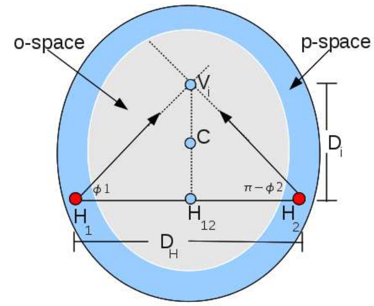


Fig. 4. Elements of the model o-space for L-Shape F-formation.

IV. The algorithm Risk-RRT

As starting point for navigation we chose the strategy proposed in [18]. This algorithm was thought to operate in dynamic, uncertain environment, it supposes that the moving pedestrians detected in the environment follow typical motion patterns that are represented by Gaussian processes which have been learned by an off-board platform before navigation and to be known by the robot. The planning algorithm is based on an extension of the Rapidly-exploring Random Tree algorithm [19], where the likelihood of the obstacles future trajectory and the probability of collision is explicitly taken into account. The tree is grown in a random fashion but a bias is included to direct the search to the goal. Best trajectory (path in the tree) is chosen using as heuristic the "probability of success" and distance to the goal of its nodes. We extended the Risk-RRT by including the knowledge of personal space of pedestrians and the possible interactions between them. The interaction we are focusing on is the conversation between two pedestrians. We penalize paths that passes in the personal space of pedestrians and in the o-space of interactions taking place in the environment by calculating a cost for each one, see eq. 13 and eq. 11. In this section, we present the partial motion planning algorithm Risk-RRT and the collision risk assessment modified in order to include our new constraints.

A. Environment model

At a given instant, the robot knowledge about the state of the world, as proposed by [18], is represented by: an estimation of the state of the robot, a set of Gaussian Processes which represent the typical patterns of the dynamic obstacles, a goal position, an occupancy grid which represents the structure of the static environment and a list of moving objects their estimated position, velocity and previous observations. To take in account the new constraints we include to the list:

1) A model of personal space $PS(o_m)$ attached to each dynamic obstacle o_m, according to section III-A
2) A list $LI = \{Z_i\}_{i=1..r}$ of interactions detected in the environment, each interaction Z_i has a model of o-space attached to it.

B. Probabilistic Risk of Collision [20] [21]

When searching for a safe path, the algorithm must determine how much is the risk of collision of taking an action $u \in U$ when in configuration $q(t_1)$. This risk can be written as $P(coll(q(t_1), u) = 1)$, the probability of collision will be referred as P_c in the rest of the paper. The risk is computed on the basis of the probability of occupation of the surface A which is swept by the robot moving from $q(t_1)$ under control u in the interval to $[t_1, t_2]$:

$$q(t_2) = f(q(t_1), u, \tau) \tag{3}$$

$$A = \iint_{t_1}^{t_2} q(t)dt \tag{4}$$

where $f(.)$ is the motion model of the robot and $\tau = t_2 - t_1$ is the time step. The risk of collision must incorporate both the static and the moving obstacles. Even when two humans in conversation don't exhibit a significant motion they must be treated as dynamic ones because they represent more risk than static obstacles. The space occupied by personal space and o-space can't be detected by sensors and for this reason this spaces are linked to the dynamic obstacles and the computation of their costs is reflected on the probability of collision of the robot with them. We make also the hypothesis that moving obstacles and static obstacles cannot overlap, and consequently that collision with a static obstacle and collision with each one of the moving obstacles are mutually exclusive events, which yields:

$$P_c = P_{cs} + (1 - P_{cs}) \cdot P_{cd} \tag{5}$$

$$P_{cd} = 1 - \prod_{m=1}^{M} [1 - P_{cd}(o_m)] \tag{6}$$

where P_{cs} is the probability of collision due to the static obstacles, $P_{cd}(o_m)$ is the probability of collision due to the dynamic obstacle o_m and P_{cd} is the probability of collision due to all the dynamic obstacles.

The static obstacles are represented in the occupancy grid which is assumed to be stationary. Given $\mathcal{M}(t_0)$ with $t_0 \le t_1$ the most recent estimation of the static map and $\varsigma \subset M(t_0)$ the subset of cells which is the minimal approximation of surface A, the risk of collision with a static obstacle is given by the max probability over the subset ς:

$$P_{cs} = \max_{\varsigma}(P(Occ(Cell_{x,y}) = 1)) \tag{7}$$

where $Cell_{x,y}$ is the cell of the occupancy grid in the (x, y) position. The risk of collision with a moving obstacle o_m is approximated by the probability that the area swept by the robot intercepts the one swept by the obstacle in the considered interval:

$$P_{cd}(o_m) = P(o_m(t) \cap A \neq \emptyset, \forall t \in [t_1, t_2]) \tag{8}$$

The prediction $o_m(t)$ is given by a weighted sum (mixture) of Gaussian Processes. A Gaussian Process is a generalization of the Gaussian probability distribution in function space, see [21] for a more detailed explanation and equations for Gaussian Processes. First, each Gaussian component k is considered separately, then all the Gaussian components are summed:

$$P_{cd}(o_m, k) = \int_A G(o_m(t), \mu_k, \Sigma_k) \tag{9}$$

$$P_{cd}(o_m) = \sum_{k=1}^{K} w_{mk} P_{cd}(o_m, k) \tag{10}$$

where $P_{cd}(o_m, k)$ is the probability of collision with the obstacle m moving along pattern k; $G(o_m(t), \mu_k, \Sigma_k)$ is the Gaussian Process representing pattern k, given the observation history of object o_m. The probability is marginalized over the set of possible patterns to yield $P_{cd}(o_m)$, where w_{mk} is the weight of the k component for object m.
In order to choose an appropriate path, the Risk-RRT uses the risk of collision of a particular action to calculate the"probability of success" of each partial path [18].

1) Adding social constraints: In this section we explain how we include the social constraints to the model before presented, being this the main contribution of the paper. First we define PZ_i as the probability of disturbing by passing inside the o-space (sec. III-B) of interaction i, and we calculate it as:

$$PZ_i = \max_{\varsigma}(\Gamma_{C_i,S_i}(Cell_{x,y})) \tag{11}$$

To reflect the fact of disturbing an interaction we think of it as a collision with a dynamic obstacle and modify the equation 6 to get:

$$P_{cd} = 1 - \prod_{m=1}^{M} [1 - P_{cd}(o_m)] \prod_{i=1}^{r} [1 - PZ_i] \tag{12}$$

In the case of the personal space we define P_{ps} as the probability of disturbing by passing in the personal space of the human o_m. We can approximate P_{ps} as the probability that A, the area swept by the robot, intercepts the one represented by the personal space:

$$P_{ps}(o_m, k) = \int_A PS(o_m(t)) \tag{13}$$

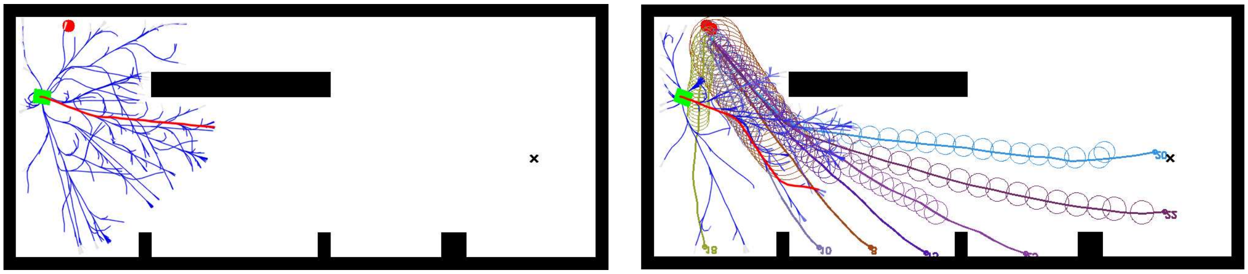
Where $PS(o_m(t)$ is the model of personal space centered in $o_m(t)$ as described in III-A. Again, to take in account this last constraint we need to modify the original equation 10 to get:

$$P_{cd}(o_m) = \sum_{k=1}^{K} w_{mk} P_{cd}(o_m, k) P_{sp}(o_m, k) \tag{14}$$

After these extensions the "probability of success" calculated for every partial path is given by the probability of not encountering a collision along the path and not entering in a personal space or an o-space.

C. The goal-oriented navigation algorithm

The goal oriented navigation proposed is described in Algorithm 1. It combines three tasks: one dedicated to perception (of static and moving obstacles), a task for planning partial but safe trajectories and a task for navigating



(a) (b)

Fig. 5. Example of execution of Risk-RRT algorithm. In a) the robot navigation system has created a tree of possible paths to follow, robot is the green rectangle, the chosen path is in red. In (b) we can observe how the robot has adapted its trajectory trying to avoid a possible collision with pedestrian (in red) by considering the predictions of typical pedestrian trajectories.

safely along planned trajectories. The prediction done for forecasting the position of moving obstacles in the near future is based on learned Gaussian Processes [18].

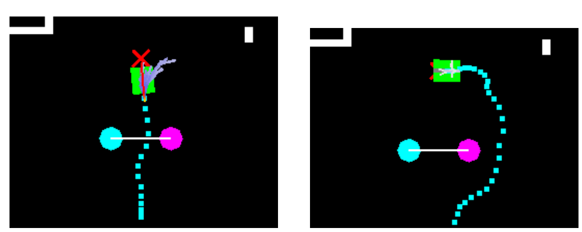
Algorithm 1 Risk-RRT

```
1: procedure RISK-RRT
2:     trajectory = empty
3:     Tree = empty
4:     Goal = read()
5:     t= clock()
6:     while Goal not reached do
7:         if trajectory is empty then
8:             brake
9:         else
10:            move along trajectory for one step
11:        end if
12:        observe (X);
13:        delete unreachable trajectories(T, X)
14:        observe(Map, movingobstacles)
15:        t= clock()
16:        predict moving obstacles at time t, ..., t + Nτ
17:        if environment different then
18:            update trajectories(T,Map,moving obstacles)
19:        end if
20:        while clock()< t + τ do
21:            grow trajectories with depth<= N in T
22:        end while
23:        trajectory = Choose best trajectory in T
24:        t = clock()
25:    end while
26:    brake
27: end procedure
```

Risk-RRT takes explicitly into account the real-time constraint and limits the time available for planning to a fixed interval. After each planning cycle, the planned trajectory is generally just a partial trajectory. Execution and planning are done in parallel: while the robot moves a step along the planned partial path, the tree is updated (line 18 of Algorithm 1) with the information coming from the perception algorithm, the tree is grown and the new partial path is passed for execution when the time step is over. In the fig. 5 we can observe an example of navigation employing Risk-RRT in the case of one pedestrian entering in the environment and robot going to its goal. At the beginning the robot has explored the environment and then decides to follow one trajectory, some steps ahead when it detects the presence of pedestrian, a prediction is realized based in the Gaussian processes and it must adjust its previous choice to avoid a collision with the human.

V. SIMULATION RESULTS

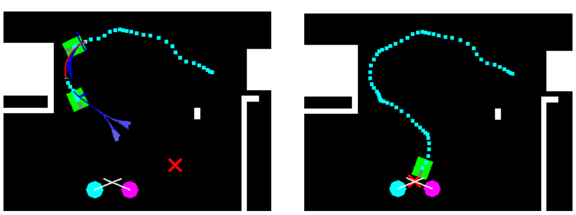
To test our models of interaction we have chosen a scenario that shows one conversation between two humans standing in a spacious area, this is because we want to decrease the effect of the structure of the environment in the management of space done by people. The simulation loads a map previously constructed by a SLAM function using a laser mounted on a wheelchair and creates an occupancy grid based on it. The pedestrians are placed in a Vis-a-Vis F-formation, that is, facing each other in theirs social zone (sec. III-A). The space between them is big enough to let the robot passing. Detecting conversation interactions is done, first by finding pedestrians that are closer than a maximum distance, then by check if their velocities are under a maximum velocity and finally taking in account the orientation of their bodies to match one of the F-formations defined in section III-B. The robot simulated is an autonomous wheelchair with two wheels, the model used is that of a differential robot system.



a) (b)

Fig. 6. Change in the behavior of the wheelchair (green). In (a) the navigation doesn't take in account the personal spaces nor the Vis-a-vis formation and chooses a path that interrupts the interaction of two humans (circles), the goal is the red cross, (b) the robot has more information and respects the social conventions of space.

The concepts exposed in section III have been implemented in a navigation algorithm taking as base our previously designed Risk-RRT approach [20], [18].
The first task was reaching the goal defined by the user, we choose an initial position for the robot and a goal location in such a way that the short distance between them passes in the middle of human positions. First, we run the algorithm original and we note that the chosen path has a tendence to interrupt the interaction, one example in fig. 6 (a), then we run the algorithm modified and we can see (fig. 6 (b)) that the tendence in the behavior of the wheelchair (in green) can be changed if we detect the interaction and reduce the probability of disturbing the conversation and the probability of disturbing by passing into the personal space of pedestrians.
As a second task, using the same scenario we let the wheelchair to explore the environment (fig. 7) and find a group in conversation to join it, this was done by choosing a random initial location for the wheelchair and random goals to reach, once that the wheelchair detect the first conversation, the new goal becomes the center of the o-space for the interaction detected. In this case we detect interactions only in a semicircular region centered in the wheelchair and oriented to the front of it. The wheelchair approaches to the group and because of the effect of interaction model it stops at p-space distance (sec. III-B), a behavior that coincides with that of a person approaching to a group and waiting for the reaction (acceptance) of the group.



a) (b)

Fig. 7. The wheelchair (green) explores its environment (a), it detects a conversation, approach to humans and stops at p-space distance (b), a behavior that can be judged social

VI. Conclusions and Future Work

The approach presented in this paper shows a way to take in account social conventions in navigation strategies providing the robot with the ability to respect the personal space and the o-space of people in its environment when moving safely towards a given goal. In the same way these models were useful to guide the robot for a "joining a group" task. The previous concepts have been implemented by extending a previously designed navigation algorithm, the Risk-RRT approach. We have shown in simulation that the behavior of a robot can be changed if we detect an interaction. Our current work aims of implementing our approach on a real autonomous system like a wheelchair and perform some experiments with real humans.
In a dynamic environment it is not enough detecting interactions because it could be too late to take a decision, we need to predict when and where an interaction will take place. Our future work will be focused in adding a technique for better predicting the creation of an o-space in the path of the robot.

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